Mathematics
Higher level
Paper 3 – discrete mathematics

Monday 8 May 2017 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

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Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 16]

(a) Use the Euclidean algorithm to find the greatest common divisor of 264 and 1365. [5]

(b) (i) Hence, or otherwise, find the general solution of the Diophantine equation

$$264x - 1365y = 3 .$$

(ii) Hence find the general solution of the Diophantine equation

$$264x - 1365y = 6 .$$ [8]

(c) By expressing each of 264 and 1365 as a product of its prime factors, determine the lowest common multiple of 264 and 1365. [3]

2. [Maximum mark: 12]

The weights of the edges in the complete graph G are given in the following table.

	A	B	C	D	E	F
A	–	4	9	8	14	6
B	4	–	1	14	9	3
C	9	1	–	5	12	2
D	8	14	5	–	11	12
E	14	9	12	11	–	7
F	6	3	2	12	7	–

(a) Starting at A, use the nearest neighbour algorithm to find an upper bound for the travelling salesman problem for G. [5]

(b) By first deleting vertex A, use the deleted vertex algorithm together with Kruskal's algorithm to find a lower bound for the travelling salesman problem for G. [7]

3. [Maximum mark: 9]

(a) In the context of graph theory, explain briefly what is meant by

(i) a circuit;

(ii) an Eulerian circuit. [2]

(b) The graph G has six vertices and an Eulerian circuit. Determine whether or not its complement G' can have an Eulerian circuit. [3]

(c) Find an example of a graph H, with five vertices, such that H and its complement H' both have an Eulerian trail but neither has an Eulerian circuit. Draw H and H' as your solution. [4]

4. [Maximum mark: 13]

Consider the recurrence relation $au_{n+2} + bu_{n+1} + cu_n = 0$, $n \in \mathbb{N}$ where a, b and c are constants. Let α and β denote the roots of the equation $ax^2 + bx + c = 0$.

(a) Verify that the recurrence relation is satisfied by

$$u_n = A\alpha^n + B\beta^n,$$

where A and B are arbitrary constants. [4]

(b) Solve the recurrence relation

$u_{n+2} - 2u_{n+1} + 5u_n = 0$ given that $u_0 = 0$ and $u_1 = 4$. [9]